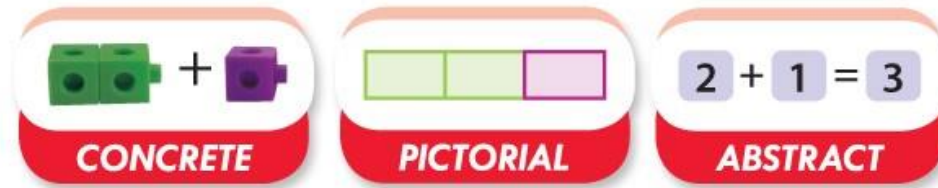


MICKLEOVER PRIMARY SCHOOL

Written Calculation Policy
2019

At Mickleover Primary, we recognise the importance of a common and progressive approach to the introduction of standard written methods, to ensure that children have secure calculation skills that are appropriate to their understanding of number. This policy outlines how written calculations are taught throughout the school based on a **Mastery Approach** that uses a **concrete, pictorial and abstract** approach to secure and deepen understanding. This approach recognises that in order for pupils to understand abstract concepts, they must first learn mathematical concepts through the use of concrete resources and pictorial representation.



Concrete is the 'doing' stage, using concrete objects to solve problems. It brings concepts to life by allowing children to handle physical objects themselves.

Pictorial is the 'seeing' stage, using representations of the objects involved in maths problems. This stage encourages children to make a mental connection between the physical object and abstract levels of understanding, by drawing or looking at pictures, circles, diagrams or models which represent the objects in the problem.

Abstract is the 'symbolic' stage, where children are able to use abstract symbols to model and solve maths problems.

As pupils progress in their maths, they become ready to handle more formal written methods that in many cases increase efficiency. However, pupils should not be moved onto these methods before their conceptual understanding of each operation is sound. Also, pupils should not be moved on automatically to the next calculation strategy - the policy should be used with professional judgement of what is appropriate for the pupils in each class. Although the focus of this policy is on pencil and paper procedures, it is important to recognise that the ability to calculate mentally lies at the heart of numeracy. Mental calculation should be seen as complementary to written recordings, as in every written method there is an element of mental processing. Supporting all calculation work should be a sound understanding of estimation and checking the reasonableness of an answer. Children should be taught to use rounding to support estimation and to check answers against the question to ensure it is reasonable and fits the real life situation (especially in the case of division and remainders).

Written ADDITION methods using a CONCRETE PICTORIAL and ABSTRACT approach.

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

Foundation 2

Concrete

Children find the total number of objects by combining two parts and counting all of them.

- Through practical activities, using fingers and through discussion they will begin to use the vocabulary involved in addition.

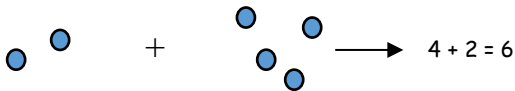


'You have five apples and I have two apples. How many apples altogether?'

- They will record pictorially then numerically
 $5 + 2 = 7$ apples

Children add 2 single digit numbers by counting on.

- Through practical activities, children to begin counting on, starting from the highest number.



- Using a number line Or numicon counting from the biggest number.

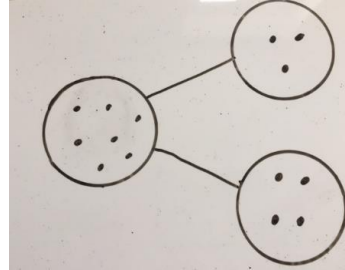


Children will find one more than a given number.

- Through songs, rhymes and practical activities children develop a sense of number.
- Children will use number line to find one more than a given number

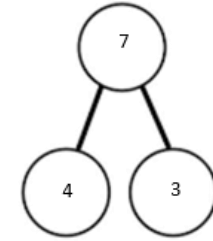
Pictorial

Children represent cubes or other objects using dots, circles crosses etc. Each part is shown on the part whole model.



Abstract

Children represent cubes or other objects using dots, circles crosses etc. Each part is shown on the part whole model.



Written ADDITION methods using a CONCRETE PICTORIAL and ABSTRACT approach.

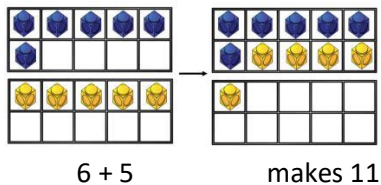
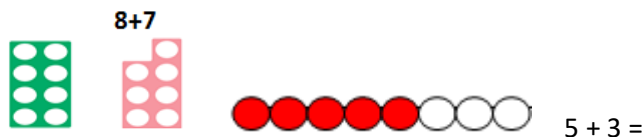
Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

Year 1

Concrete

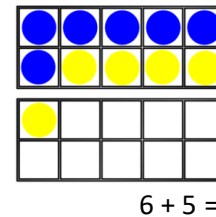
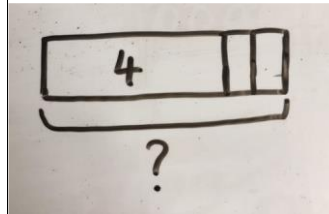
Children, read, write and interpret mathematical statements involving addition (+) and the equals (=).

- Through practical activities, using rods, cubes, numicom, number beads, number lines and 100 squares.



Pictorial

- Using a bar model or tens frame to represent the addition



Abstract

- Using an abstract numberline (in head)

What is 2 more than 4
 What is the sum of 2 and 4
 What is the total of 2 and 4

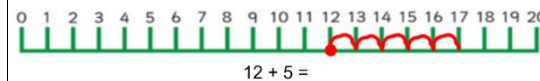


- using number bonds and related addition facts within 20 which have been learned.

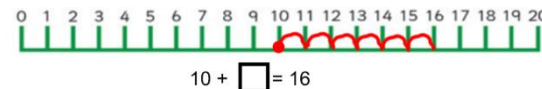
Children add one-digit and two-digit numbers within 20, including zero

Using practical equipment children combine groups, counting from the largest

- Using a number line to add two numbers together, encouraging children to start from the largest number.




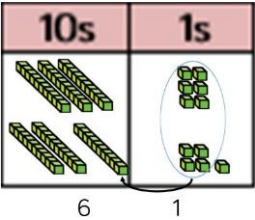
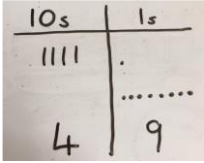
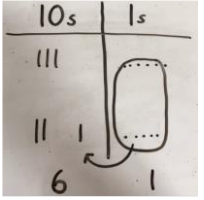
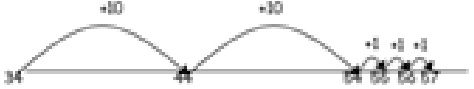
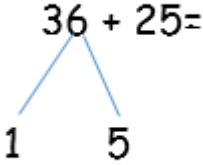
- Children solve missing number problems by counting on from the given number. eg 10 + ___ = 16



Written ADDITION methods using a CONCRETE PICTORIAL and ABSTRACT approach.

Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

Year 2

Concrete	Pictorial	Abstract				
<p>Children solve problems with addition using concrete objects as used in foundation stage 2 and Year 1.</p> <p>Children will learn to add -:</p> <ol style="list-style-type: none"> 1. A two digit number and ones 2. A two digit number and tens 3. Two two-digit numbers 4. Three one-digit numbers <ul style="list-style-type: none"> • Using Tens and Ones apparatus children add by combining groups, counting from the largest. (TO + O and TO +TO base 10 with no exchange) <div style="text-align: center;">  </div> <ul style="list-style-type: none"> • Using Tens and Ones apparatus children add by combining groups, where 10 ones are exchanged for a Ten. <div style="text-align: center;">  </div>	<p>Children solve problems with addition using pictorial representations as used in foundation stage 2 and Year 1.</p> <ul style="list-style-type: none"> • Using place value knowledge children combine <u>Tens</u> and <u>Ones</u> to add. <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">  </div> <div style="text-align: center;">  </div> </div> <ul style="list-style-type: none"> • Using an empty number line to add two-digit numbers. <div style="text-align: center;"> <p>$34 + 23 = 57$</p>  </div> <ul style="list-style-type: none"> • Bar models are used to show pictorial representations <div style="text-align: center;"> <table border="1" style="margin: auto;"> <tr> <td colspan="2" style="text-align: center;">?</td> </tr> <tr> <td style="width: 50px; text-align: center;">21</td> <td style="width: 50px; text-align: center;">34</td> </tr> </table> </div>	?		21	34	<ul style="list-style-type: none"> • Using place value knowledge children combine <u>Tens</u> and <u>Ones</u> to add in head. <div style="display: flex; align-items: center; justify-content: center;"> <div style="text-align: center; margin-right: 20px;">  </div> <div> <p>$36 + 25 =$</p> <p>$30 + 20 = 50$</p> <p>$5 + 5 = 10$</p> <p>$50 + 10 + 1 = 61$</p> </div> </div>
?						
21	34					

Written ADDITION methods using a CONCRETE PICTORIAL and ABSTRACT approach.

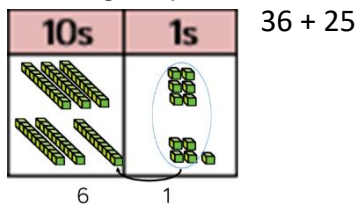
Key language: sum, total, parts and wholes, plus, add, altogether, more, 'is equal to' 'is the same as'.

Year 3

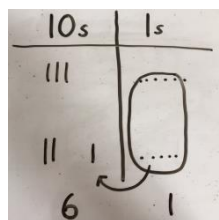
Year 5/ 6

Add numbers with up to three digits, using formal written method of columnar addition

TO + TO using base 10.
Continue to develop understanding of partitioning and place value



Children to represent the base 10 in a place value chart.



Looking for ways to make 10.

$$36 + 25 = 30 + 20 = 50$$

$$5 + 5 = 10$$

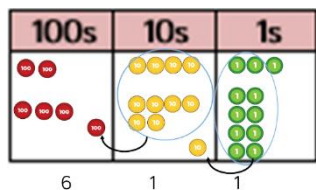
$$50 + 10 + 1 = 61$$

1 5 36

Formal method:

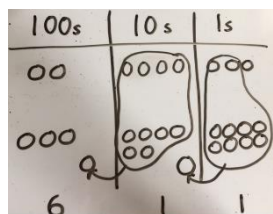
$$\begin{array}{r} +25 \\ 36 \\ \hline 61 \\ 1 \end{array}$$

Use of place value counters or Dienes to add HTO + TO, HTO + HTO etc



When there are 10 ones in the 1s

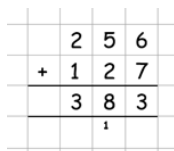
column- we exchange for 1 ten, when there are 10 tens in the 10s column- we exchange for 1 hundred



$$\begin{array}{r} 243 \\ +368 \\ \hline 611 \\ 1 \quad 1 \end{array}$$

YEAR 4

Add numbers with up to 4 digits using the formal written methods of columnar addition building on concrete pictorial abstract approach of year 3



$$\begin{array}{r} 783 \\ + 42 \\ \hline 825 \\ \pm \end{array}$$

$$\begin{array}{r} 625 \\ + 48 \\ \hline 673 \\ \pm \end{array}$$

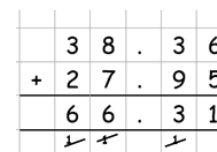
Carry below the line and cross off when added into the calculation

Add whole numbers with more than 4 digits, including using formal methods



$$\begin{array}{r} 23587 \\ + 1475 \\ \hline 25062 \\ \hline 50124 \\ \hline 121 \end{array}$$

Begin to add two or more decimal fractions with up to three digits and the same number of decimal places; know that decimal points should line up under each other, particularly when adding or subtracting mixed amounts, e.g. 3.2 m - 280 cm.



When working with decimals, always make each number have the same amount of digits to the right of the decimal point

3.7 + 2.35

$$\begin{array}{r} 3.70 \\ + 2.35 \\ \hline 6.05 \end{array}$$

In year 6, there is an expectation that children will continue to practise and use the formal written method for larger numbers and decimals and use these methods when solving problems, when appropriate

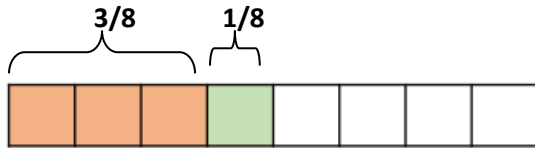
Addition of fractions

Year 3

Year 4

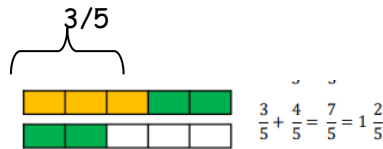
Year 5

Use bar models to add fractions with the same denominator within one whole

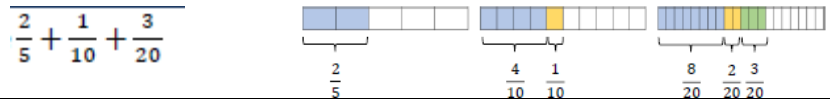
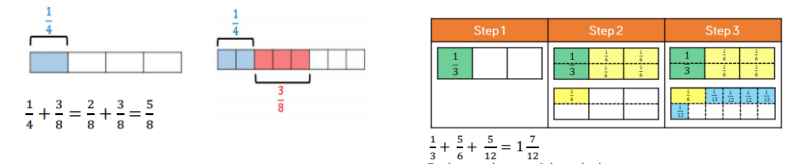
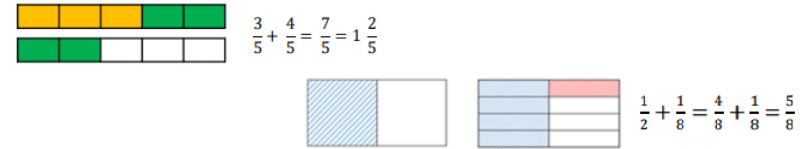


We can use this model to calculate $\frac{3}{8} + \frac{1}{8} = \frac{4}{8}$

Use bar models to add fractions of the same denominators



Use bar models to add fractions with same denominators and multiples of the same number

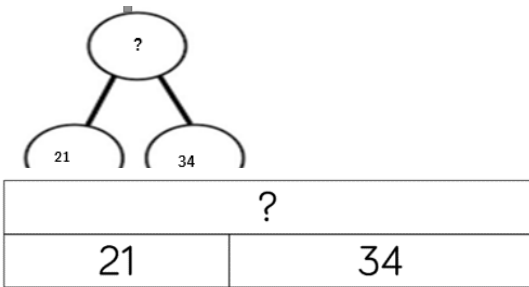


Year 6

$$2\frac{1}{4} + 1\frac{1}{6} = 1\frac{3}{12} + 1\frac{2}{12} = 3\frac{5}{12}$$

Pupils should add fractions with different denominators and mixed numbers using the concept of equivalent fractions. Leading to abstract method of using common denominators

Conceptual Variation; different ways to ask children to solve 21 + 34



Word problems: In year 3, there are 21 children and in year 4, there are 34 children. How many children in total?

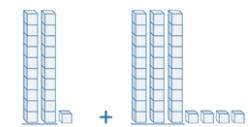
$$21 + 34 = 55. \text{ Prove it}$$

$$\begin{array}{r} 21 \\ +34 \\ \hline \end{array}$$

$$21 + 34 =$$

$$\boxed{} + 34$$

Calculate the sum of twenty-one and thirty-four



10s	1s
10 10	1
10 10 10	?
?	5

Missing digit calculations

$$2\boxed{} + \boxed{}4 = 55$$

Written SUBTRACTION methods using a CONCRETE PICTORIAL AND ABSTRACT approach

Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

Foundation 2

Concrete

Children will engage in a variety of counting songs, rhymes and practical activities to develop a sense of number.

Children will find one less than a given number.

- In practical activities, using objects and fingers they will begin to use the vocabulary involved in subtraction



'You have five apples and I eat one apple. How many apples left?'

- They will record pictorially then numerically
 $5 - 1 = 4$ apples

Children subtract from 2 single digit numbers, by counting back to find the answer practically.

- Using objects then pictures, children subtract a single digit number



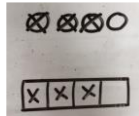
5 subtract 3 = 2

- Using Numicon children represent the subtraction by taking away covering the number.

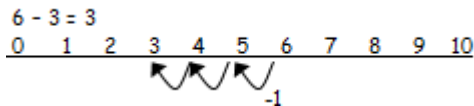


Pictorial

- Children draw the concrete resources and cross out the correct amount. Bar models are also used.

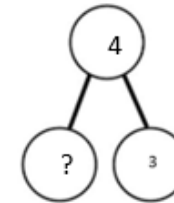


- Using a number line children count back below the line to show subtraction.



Abstract

- Part Whole models show the equation for children to find the correct answer



Written SUBTRACTION methods using a CONCRETE PICTORIAL AND ABSTRACT approach


Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

Year 1

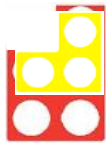
Concrete

Children read, write & interpret mathematical statements involving subtraction (-) & equals (=).

- Through practical activities, using rods, cubes, numicom, number beads, number lines and 100 squares.



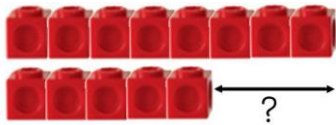
$$5 - 3 = 2$$



Children subtract one-digit & two-digit numbers to 20, including zero.

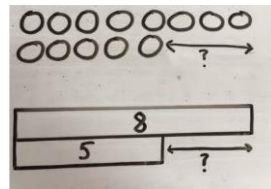
- Children find the difference using subtraction.

Calculate the difference between 8 and 5.

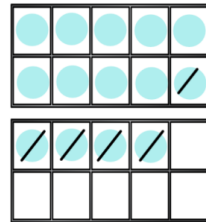


Pictorial

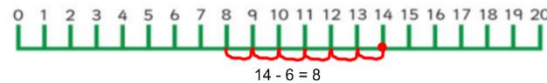
- Children draw cubes/counters and represent subtraction on bar models.



- Children present the ten frame pictorially and discuss what they did to make 10.



Children use a number line to subtract a number, counting back below the line.



Abstract

- Children use number bonds and related addition facts within 20 which have been learned.

Written SUBTRACTION methods using a CONCRETE PICTORIAL AND ABSTRACT approach

Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

Year 2

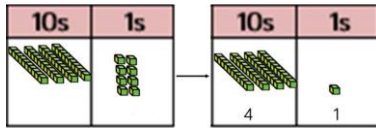
Concrete

Children subtract numbers using concrete objects and pictorial representations as used in foundation and year 1.

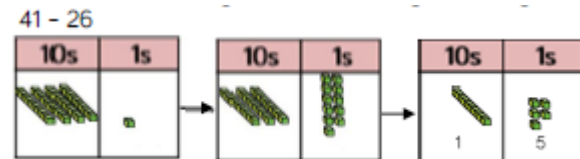
Children will learn to subtract -:

- A two digit number and ones
- A two digit number and tens
- Two two-digit numbers

- Using place value knowledge children subtract Tens and Ones. (No exchange)
48 - 7 =

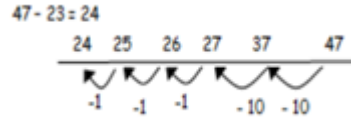


- Using Tens and Ones apparatus children subtract by exchanging ten ones for a Ten .

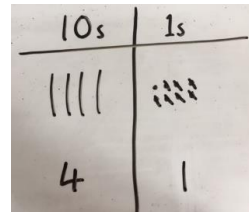


Pictorial

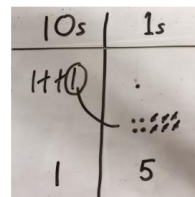
- Using place value knowledge, children subtract using a number line



- Using place value knowledge, children subtract Tens and Ones. (No exchange)
48 - 7 =



- Using Tens and Ones apparatus children subtract by exchanging ten ones for a Ten



Abstract

- Using knowledge of addition and subtraction families and the inverse relationship of addition and subtraction.

$$\begin{array}{l} 3 + 2 = 5 \quad 2 + 3 = 5 \quad \text{Number} \\ 5 - 2 = 3 \quad 5 - 3 = 2 \quad \text{families} \end{array}$$

- Using Inverse knowledge

$$48 + 36 = 84 \quad \text{so} \quad 84 - 36 = 48$$

- Use partitioning of subtrahend before subtraction

$$\begin{array}{c} 14 - 5 = 9 \\ \swarrow \quad \searrow \\ 4 \quad \quad 1 \end{array}$$

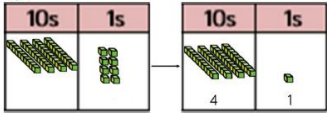
Written SUBTRACTION methods using a CONCRETE PICTORIAL AND ABSTRACT approach

Key language: take away, less than, the difference, subtract, minus, fewer, decrease.

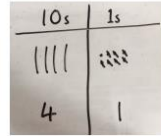
Year 3

Subtract numbers with up to three digits, using formal written method of columnar subtraction

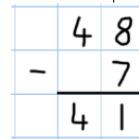
Column method using base 10.
48-7



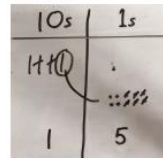
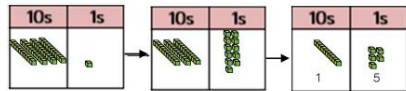
Children to represent the base 10 pictorially.



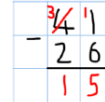
Column method or children could count back 7]



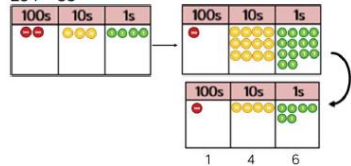
Column method using base 10 and having to exchange.
41 - 26



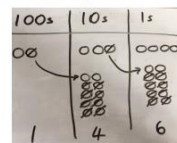
Formal column method. Children must understand that when they have exchanged the 10 they still have 41 because $41 = 30 + 11$.



Column method using place value counters.
234 - 88



Represent the place value counters pictorially; remembering to show what has been exchanged.



Formal column method. Children must understand what has happened when they have crossed out digits.

$$\begin{array}{r} \overset{2}{2} \overset{1}{3} 4 \\ - 88 \\ \hline 156 \end{array}$$

Year 5

Subtract whole numbers with more than 4 digits, including using formal written method (columnar subtraction)

the Use compact ('decomposition') method and the vocabulary 'exchange' to include decimals

8 ⁷	16	.	3 ²	10	
-	1	9	.	0	4
<hr/>					
6	7	.	2	6	

In this example, we have 'exchanged' one of the three tenths for ten hundredths

Year 6 as per Year 5 using larger number

Year 4

Subtract numbers with up to 4 digits using the formal written methods of columnar subtraction where appropriate

$$\begin{array}{r} 6 \text{ } 14 \text{ } 1 \\ \text{7} \text{ } 5 \text{ } 4 \\ - 86 \\ \hline 668 \end{array}$$

	5	6 ⁵	12
-	3	1	9
<hr/>			
	2	4	3


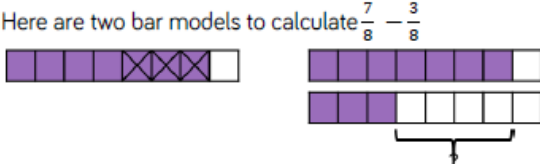


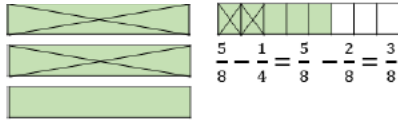
NOTE: In both examples, we have 'exchanged' one of the six tens for ten ones.

Children will also begin to find the difference between two three-digit sums of money, with or without 'adjustment' from the pence to the pounds; know that decimal points should line up under each other

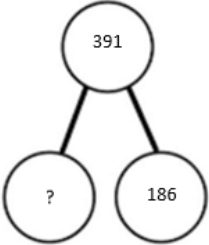
£3.50- £1.67

$$\begin{array}{r} 1.67 \\ - 0.30 \\ \hline 1.37 \end{array}$$

Subtraction of fractions

Year 3	Year 4	Year 5	Year 6
<p>Subtract fractions with the same denominator within one whole</p> <p style="text-align: center;">$5/7 - 2/7 = 3/7$</p> 	<p>Subtract fractions with the same denominator</p> <p>Here are two bar models to calculate $\frac{7}{8} - \frac{3}{8}$</p> 	<p>Subtract fractions with the same denominator and multiples of the same number</p> <p style="text-align: center;">$\frac{1}{3} - \frac{1}{12} = \frac{3}{12}$</p>   <p style="text-align: center;">$1\frac{3}{4} - \frac{5}{8} = 1\frac{1}{8}$</p> <p style="text-align: center;">$3\frac{5}{8} - 2\frac{1}{4} = 1\frac{3}{8}$</p>  <p style="text-align: center;">$\frac{5}{8} - \frac{1}{4} = \frac{5}{8} - \frac{2}{8} = \frac{3}{8}$</p> <p style="text-align: center;">$3 - 2 = 1$</p>	<p>Subtract fractions with different denominators</p> <p>Building on Y5 methods to abstract methods by finding common denominator</p> <p style="text-align: center;">$\frac{3}{4} - \frac{2}{3} = \frac{9-8}{12} = \frac{1}{12}$</p>

Conceptual variation; different ways to ask children to solve 391 - 186

 <table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 150px; text-align: right;">391</td> <td></td> </tr> <tr> <td style="width: 100px; text-align: right;">186</td> <td style="width: 50px; text-align: center;">?</td> </tr> </table>	391		186	?	<p>Raj spent £391, Timmy spent £186. How much more did Raj spend?</p> <p>Calculate the difference between 391 and 186</p>	<p style="text-align: center;"><input style="width: 40px; height: 30px;" type="text"/> = 391-186</p> <table style="margin-left: auto; margin-right: auto;"> <tr> <td style="text-align: right; padding-right: 10px;">391</td> <td style="padding-right: 10px;">What is 186 less than 391?</td> </tr> <tr> <td style="text-align: right; padding-right: 10px;"><u>-186</u></td> <td></td> </tr> <tr> <td style="text-align: right; padding-right: 10px;">_____</td> <td></td> </tr> </table>	391	What is 186 less than 391?	<u>-186</u>		_____		<p style="text-align: center;">Missing digit calculations</p> <div style="text-align: center;"> <table style="margin: auto;"> <tr> <td></td> <td style="text-align: center;">3</td> <td style="text-align: center;">9</td> <td style="text-align: center;"><input style="width: 20px; height: 20px;" type="text"/></td> </tr> <tr> <td style="text-align: center;">-</td> <td style="text-align: center;"><input style="width: 20px; height: 20px;" type="text"/></td> <td style="text-align: center;"><input style="width: 20px; height: 20px;" type="text"/></td> <td style="text-align: center;">6</td> </tr> <tr> <td colspan="4" style="text-align: center;">-----</td> </tr> <tr> <td></td> <td style="text-align: center;"><input style="width: 20px; height: 20px;" type="text"/></td> <td style="text-align: center;">0</td> <td style="text-align: center;">5</td> </tr> </table> </div>		3	9	<input style="width: 20px; height: 20px;" type="text"/>	-	<input style="width: 20px; height: 20px;" type="text"/>	<input style="width: 20px; height: 20px;" type="text"/>	6	-----					<input style="width: 20px; height: 20px;" type="text"/>	0	5
391																													
186	?																												
391	What is 186 less than 391?																												
<u>-186</u>																													

	3	9	<input style="width: 20px; height: 20px;" type="text"/>																										
-	<input style="width: 20px; height: 20px;" type="text"/>	<input style="width: 20px; height: 20px;" type="text"/>	6																										

	<input style="width: 20px; height: 20px;" type="text"/>	0	5																										

Written MULTIPLICATION methods using a CONCRETE, PICTORIAL and ABSTRACT approach

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Foundation 2

Concrete

Children solve problems involving doubling.

- In practical activities and through discussion children will begin to use the vocabulary of multiplication - groups, lots, double.
- Through practical activities solve problems including doubling.

Pictorial

Children solve problems involving doubling.

They will record pictorially -:



$$3 + 3 = 6 \text{ lollies}$$

'You have 3 lollies and your friend gives you 3 more.
How many do you have altogether?

Abstract

Children solve problems involving doubling.

They will record numerically -:

$$3 + 3 = 6 \text{ lollies}$$

Double 3 is 6

Written MULTIPLICATION methods using a CONCRETE, PICTORIAL and ABSTRACT approach

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Year 1

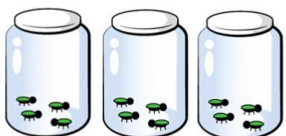
Concrete

Children solve one-step problems involving multiplication using concrete objects, pictorial representations

- Children count in 2's, 5's and 10's.
- Children continue to use the vocabulary of multiplication - groups, lots, double.
- Children recognise doubling as adding the same number again.



Children will put objects and pictures into repeated groups to count.



$$3 \times 4$$

$$4 + 4 + 4$$

There are 3 equal groups, with 4 in each group.



Pictorial

Children solve one-step problems involving multiplication using concrete objects, pictorial representations

- Children represent the practical resources in a picture or use a bar model

Abstract

Children solve one-step problems involving multiplication using concrete objects, pictorial representations

- Children write multiplication as a stem sentence.

$$3 \text{ groups of } 4 = 12$$

Written MULTIPLICATION methods using a CONCRETE, PICTORIAL and ABSTRACT approach

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Year 2

Concrete

Children solve problems with multiplication using concrete objects as used in foundation stage 2 and Year 1.

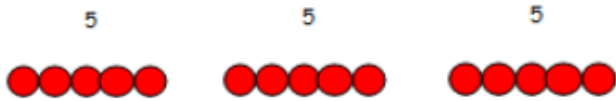
Children solve multiplication problems practically, using concrete objects and arrays

- Children solve multiplication calculations practically through repeated addition.

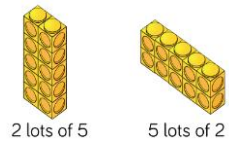
$$5 + 5 + 5$$

$$3 \times 5 = 15$$

3 groups of 5 = 15



- Children use arrays to solve multiplication calculations and illustrate commutativity.



Pictorial

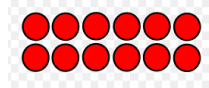
Children solve problems with multiplication using pictorial representations as used in foundation stage 2 and Year 1.

Children solve multiplication problems, pictorially using 100 squares, arrays and numberlines.

- Using a 100 square to find and discuss patterns when counting.

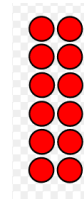
4	5	6	7	8	9	10
14	15	16	17	18	19	20

- Children draw dots to represent arrays.

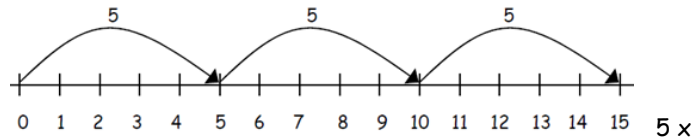


$2 \times 6 =$

$6 \times 2 =$



- Number lines are used to show multiplication as repeat addition.



$3 \times 5 = 5 + 5 + 5$

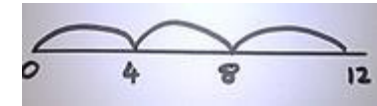
Children record calculations using \times and $=$.

Abstract

Children solve multiplication problems, using repeated addition and multiplication and division facts.

- Children recall & use multiplication facts for 2, 5 & 10 tables, including recognising odd and even numbers.

- Children use Abstract numberlines to solve multiplication problems.

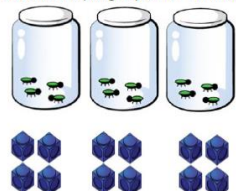
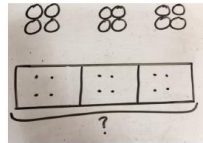
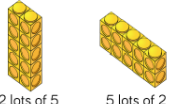
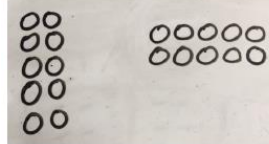


Written MULTIPLICATION methods using a CONCRETE, PICTORIAL and ABSTRACT approach

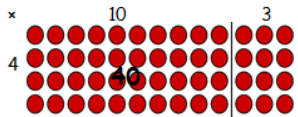
Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Year 3/4

Write and calculate statements for \times and \div using tables they know, including for $TU \times U$ using mental and progressing to formal written methods.

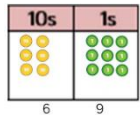
<p>Repeated grouping/repeated addition 3×4 $4 + 4 + 4$ There are 3 equal groups, with 4 in each group.</p> 	<p>Children to represent the practical resources in a picture and use a bar model.</p> 	<p>$3 \times 4 = 12$ $4 + 4 + 4 = 12$</p>
<p>Use arrays to illustrate commutativity counters and other objects can also be used. $2 \times 5 = 5 \times 2$</p> 	<p>Children to represent the arrays pictorially.</p> 	<p>Children to be able to use an array to write a range of calculations e.g. $10 = 2 \times 5$ $2 + 2 + 2 + 2 + 2 = 10$ $10 = 5 + 5$ $5 \times 2 = 10$</p>

$13 \times 4 = (10 \times 4) + (3 \times 4)$

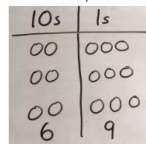


Multiply TU \times U

Formal column method with place value counters (base 10 can also be used.) 3×23



Children to represent the counters pictorially.



Children to record what it is they are doing to show understanding.

3×23 $3 \times 20 = 60$
 $20 \ 3$ $3 \times 3 = 9$
 $60 + 9 = 69$

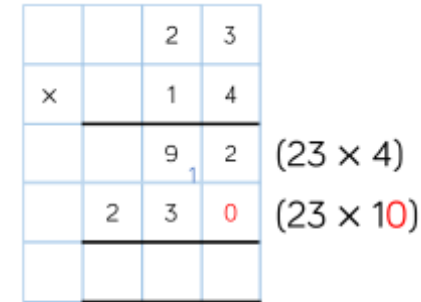
$$\begin{array}{r} 23 \\ \times 3 \\ \hline 9 \\ 60 \\ \hline 69 \end{array}$$

Year 5/ 6

Multiply numbers up to 4 digits by a one or two-digit number using a formal method, including long multiplication for two-digit numbers

When children start to multiply a $3d \times 3d$ and $4 \times 2d$, they should be confident with the abstract.

Begin with expanded if needed



Leading to

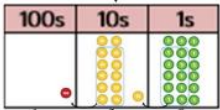
$$\begin{array}{r} 1 \ 2 \\ 1 \ 2 \ 4 \\ \times \quad 2 \ 6 \\ \hline 7 \ 4 \ 4 \\ 2 \ 4 \ 8 \ 0 \\ \hline 3 \ 2 \ 2 \ 4 \\ \pm \ \pm \end{array}$$

To get 744 children have solved 6×124
 To get 2480 children have solved 20×124

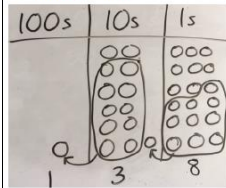
NB Carry above for \times below for addition

Formal column method with place value counters.

6 x 23



Children to represent the counters/base 10, pictorially e.g. the image below.

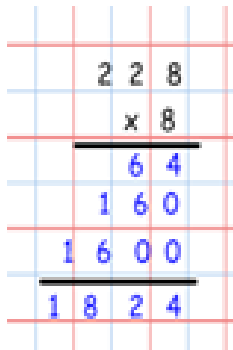


Formal written method

6 x 23 =

$$\begin{array}{r} 23 \\ \times 6 \\ \hline 18 \\ 120 \\ \hline 138 \end{array}$$

Multiply two-digit and three-digit numbers by a one-digit number using formal written



$$\begin{array}{r} 228 \\ \times 8 \\ \hline 1824 \end{array}$$

Carrying above to avoid confusion when multiplying 3 digit by 2 digit

Multiplication of fractions

Year 5

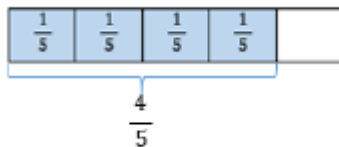
- Multiply proper fractions by whole number,

- Begin with repeated addition using models

$$\frac{1}{6} \times 4 = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}$$



- Using single bar model



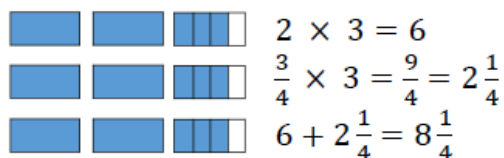
Multiply mixed numbers by whole number

$$2\frac{2}{3} \times 4$$

using repeated addition

$$2\frac{2}{3} \times 4 = 2\frac{2}{3} + 2\frac{2}{3} + 2\frac{2}{3} + 2\frac{2}{3} = 8\frac{8}{3} = 10\frac{2}{3}$$

- By partitioning

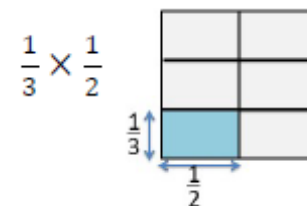


- By converting to an improper fraction

$$1\frac{5}{6} \times 3 = \frac{11}{6} \times 3 = \frac{33}{6} = 5\frac{3}{6} = 5\frac{1}{2}$$

Year 6

- Multiply simple pairs of proper fractions, writing the answer in its simplest form [for example $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$]



$$\frac{2}{3} \times \frac{3}{4}$$

First, separate the square into 3 equal parts vertically and shade 2 parts to indicate $\frac{2}{3}$.



Now separate the square into 4 equal parts horizontally and shade 3 of them to show $\frac{3}{4}$.



Since we are looking for $\frac{2}{3} \times \frac{3}{4}$ or $\frac{2}{3}$ of $\frac{3}{4}$, we

get 6 out of 12 parts that are double-shaded and represent $\frac{2}{3} \times \frac{3}{4}$.



Conceptual variation; different ways to ask children to solve 6×23

23	23	23	23	23	23
----	----	----	----	----	----

?

Mai had to swim 23 lengths, 6 times a week.

How many lengths did she swim?

With counters, prove that $6 \times 23 = 138$

Find the product of 6 and 23


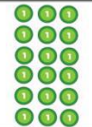
$$6 \times 23 =$$

$$= 6 \times 23$$

$$\begin{array}{r} 6 \quad 23 \\ \times \underline{23} \quad \times \underline{6} \\ \hline \hline \end{array}$$

What is the calculation?

What is the product?

100s	10s	1s
		

Written Division methods using a CONCRETE, PICTORIAL and ABSTRACT approach

Key language: double, times, multiplied by, the product of, groups of, lots of, equal groups.

Foundation 2

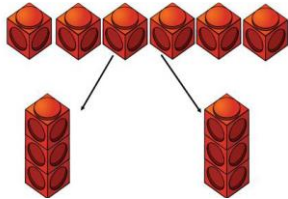
Concrete

Children solve problems involving simple halving and sharing

- In practical activities, using objects they will begin to use division vocabulary - groups, sharing.

'You have 6 buns and give your friend half. How many do you each have?'

They will record pictorially.



Pictorial

Children solve problems involving simple halving and sharing

- Using pictures and through discussion they will begin to use division vocabulary - groups, sharing.



Abstract

Children solve problems involving simple halving and sharing

- Through discussion they will begin to use division vocabulary - groups, sharing.

Written DIVISION methods using a CONCRETE, PICTORIAL and ABSTRACT approach

Key language: share, group, divide, divided by, half.

Year 1

Concrete

Children solve problems with division using concrete objects as used in foundation stage 2.

Children solve problems involving division using concrete objects.

- Through practical activities children will find half and then a quarter by sharing.

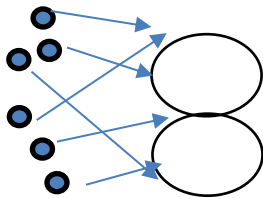
4 cakes shared between 2 people. How many do they get each?



Children use objects to group and share amounts to develop an understanding of division in a practical sense.

- Sharing - Children will have practical opportunities to share out by giving one to each plate.

E.g. 6 sweets are shared between 2 people.
How many do they have each?

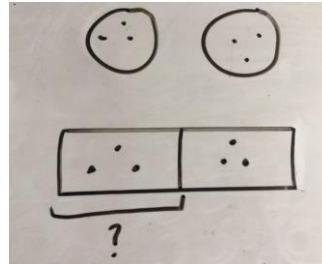


- Grouping - Children will have practical opportunities to put objects into groups of a specific number.

Pictorial

Children solve problems with addition using pictorial representations as used in foundation stage 2.

Children solve problems involving division using pictorial representations.



Children will use jottings to record both sharing equally and grouping. Then they begin to use the ÷ sign to record their division problems.

$$6 \div 2 = 3$$



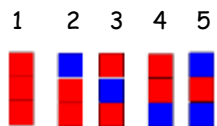
Abstract

Children count on and back from different numbers in 1s and then in multiples of 2, 5 and 10.

E.g. There are 6 sweets. How many people can have 2 sweets each?



If you have 15 cubes. How many towers of 3 cubes can you make?



Written DIVISION methods using a CONCRETE, PICTORIAL and ABSTRACT approach

Key language: share, group, divide, divided by, half.

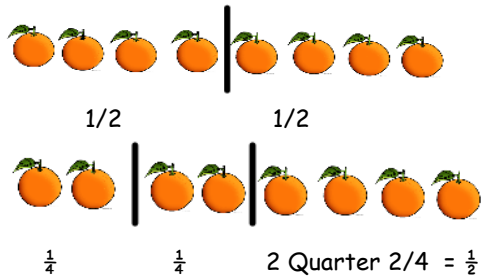
Year 2

Concrete

Children solve problems with division using concrete objects as used in foundation stage 2 and Year 1.

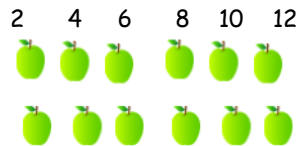
Children find a half, a quarter, a third and three quarters of shapes, objects and numbers.

- Using and sharing objects



Children continue to use grouping and sharing for division using practical apparatus.

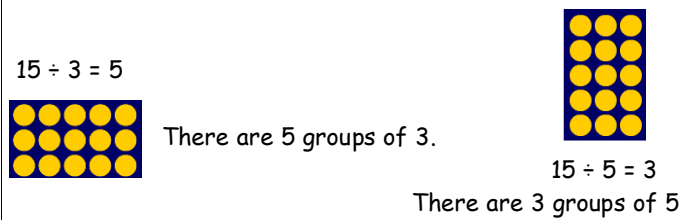
- Division facts - Children count regularly, on and back, in steps of 2, 5 and 10 using concrete objects.



Pictorial

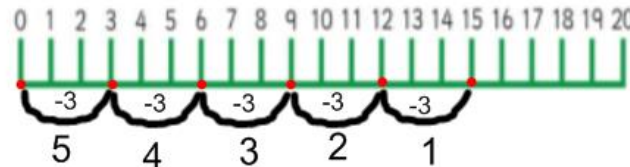
Children solve problems with addition using pictorial representations as used odd in foundation stage 2 and Year 1.

- Arrays - Children will be introduced to arrays as a pictorial representation to show division.



E.g. 15 pencils shared between 3 pots, how many in each pot?

- Repeat subtraction - Children recognise division as repeat subtraction. Using a numberline children start with the total amount to be divided (the first number). They then jump back in steps of the divisor (the second number) until they reach 0. By counting the number of steps taken we find the answer.



$$15 \div 3 = 5$$

NB. We always count backwards below the line for subtraction.

Abstract

Children recognise odd and even numbers and recall division facts for the 2, 5 and 10 multiplication tables.

E.g. Sort these numbers into and even 15, 27, 34, 75, 82

- Mental methods, and division facts - Children count regularly, on and back, in steps of 2, 5 and 10.

Children calculate mathematical statements for division within the multiplication tables Of 2, 5 and 10 and write them using division (\div) and equals ($=$) signs.

$$20 \div 5 =$$

Children partition tens and ones with larger numbers to find half, a quarter and three quarters

Find half of 48

$$48 = 40 + 8$$

$$\text{Half of } 40 = 20$$

$$\text{Half of } 8 = 4$$

$$\text{Half of } 48 = 20 + 4 = 24$$

Written DIVISION methods using a CONCRETE, PICTORIAL and ABSTRACT approach

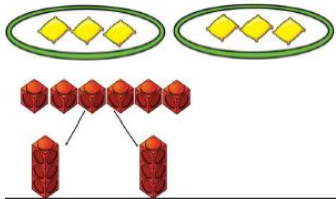
Key language: share, group, divide, divided by, half.

Year 3/ Year 4

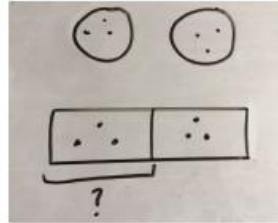
Division with a remainder-using lollipop sticks, times tables facts and repeated subtraction.

2d divided by 1d using base 10 or place value counters

Sharing using a range of objects.
 $6 \div 2$



Represent the sharing pictorially.



$$6 \div 2 = 3$$

3	3
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Children should also be encouraged to use their 2 times tables facts.

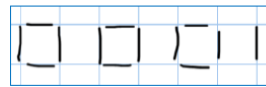
2d ÷ 1d with remainders using lollipop sticks. Cuisenaire rods, above a ruler can also be used.
 $13 \div 4$

Use of lollipop sticks to form wholes- squares are made because we are dividing by 4.



There are 3 whole squares, with 1 left over.

Children to represent the lollipop sticks pictorially.

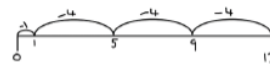


There are 3 whole squares, with 1 left over.

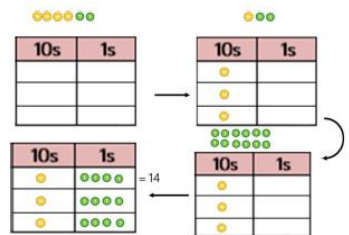
$$13 \div 4 = 3 \text{ remainder } 1$$

Children should be encouraged to use their times table facts; they could also represent repeated addition on a number line.

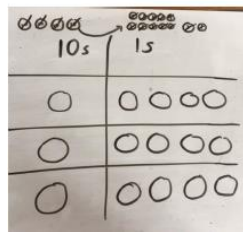
'3 groups of 4, with 1 left over'



Sharing using place value counters.
 $42 \div 3 = 14$



Children to represent the place value counters pictorially.



Children to be able to make sense of the place value counters and write calculations to show the process.

$$\begin{aligned} 42 \div 3 \\ 42 &= 30 + 12 \\ 30 \div 3 &= 10 \\ 12 \div 3 &= 4 \\ 10 + 4 &= 14 \end{aligned}$$

Division with one digit to include remainders

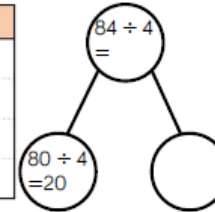
$$84 \div 4$$



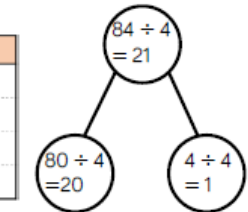
First divide the tens

Then divide the ones

Tens	Ones
10	
10	
10	
10	

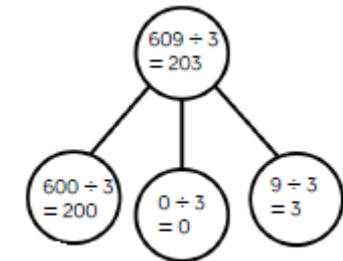


Tens	Ones
10	1
10	1
10	1
10	1

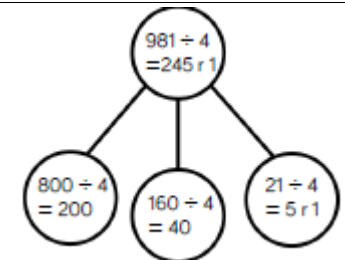


$$603 \div 3$$

Hundreds	Tens	Ones
100		1
100		1
100		1



Hundreds	Tens	Ones
100	10	1
100	10	1
100	10	1



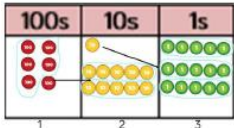
Short division (up to 3 digits by 1 digit- concrete and pictorial) See Y5 starting with 2 digit numbers beyond table facts

Year 5

Short division

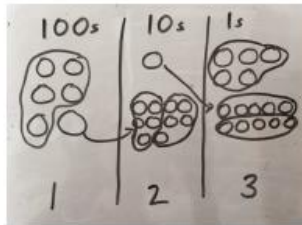
(up to 4 digits by a 1 digit number including remainders)

Short division using place value counters to group.
615 ÷ 5



1. Make 615 with place value counters.
2. How many groups of 5 hundreds can you make with 6 hundred counters?
3. Exchange 1 hundred for 10 tens.
4. How many groups of 5 tens can you make with 11 ten counters?
5. Exchange 1 ten for 10 ones.
6. How many groups of 5 ones can you make with 15 ones?

Represent the place value counters pictorially.



Children to the calculation using the short division scaffold.

$$\begin{array}{r} 123 \\ 5 \overline{) 615} \\ \underline{5} \\ 11 \\ \underline{10} \\ 15 \\ \underline{15} \\ 0 \end{array}$$

$6096 \div 8 = 762$

$3018 \div 8 = 377 \text{ R } 2 \text{ or } 377 \text{ R } 2/8$

R2 or

	0	7	6	2
8	6	60	49	16

	0	3	7	7
8	3	30	61	58

Leading to remainder with decimals

	0	3	7	7	.	2	5
8	3	30	61	58	.	20	40

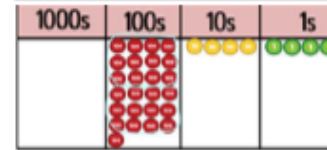
Year 6

Divide numbers up to four-digit by two-digit number using the short ('bus stop') method

Long division using place value counters
2544 ÷ 12



We can't group 2 thousands into groups of 12 so will exchange them.



We can group 24 hundreds into groups of 12 which leaves with 1 hundred.

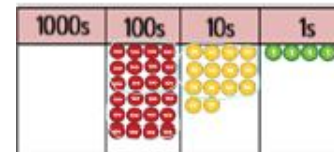
$$\begin{array}{r} 02 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

Long division for 2 digit numbers

List multiples 1st= 18

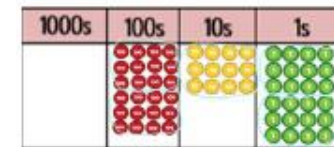
$$\begin{array}{r} 343 \\ 18 \overline{) 6174} \\ \underline{54} \\ 77 \\ \underline{72} \\ 54 \end{array}$$

5th=90



After exchanging the hundred, we have 14 tens. We can group 12 tens into a group of 12, which leaves 2 tens.

$$\begin{array}{r} 021 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 2 \end{array}$$



After exchanging the 2 tens, we have 24 ones. We can group 24 ones into 2 group of 12, which leaves no remainder.

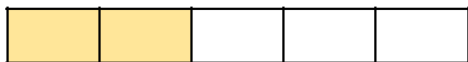
$$\begin{array}{r} 0212 \\ 12 \overline{) 2544} \\ \underline{24} \\ 14 \\ \underline{12} \\ 24 \\ \underline{24} \\ 0 \end{array}$$

10th=180

Division of fractions

Year 6

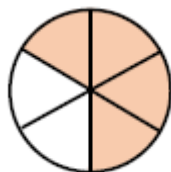
Begin with fractions where numerator is divisible by divisor



$2/3$ divided by $2 = 1/3$

$$\frac{3}{4} \div 3$$

$$\frac{4}{6} \div 2$$



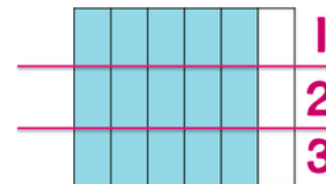
Divide a fraction by a whole number when the numerator is not divisible by the divisor $3/4 \div 5 = 3/20$

Dividing Fractions by Whole Numbers
Visual Model

$$\frac{5}{6} \div 3$$



\div



Finding your answer:

Numerator:

The shaded part of ONE of the groups formed by dividing
Each of the three groups (orange, green, yellow) has 5 shaded parts

Denominator:

Total number of equal parts
There are 3 groups of 6, equalling 18 total parts

Therefore, $5/6$ divided by 3 equals $5/18$.

Conceptual variation; different ways to ask children to solve $615 \div 5$

Using the part whole model below, how can you divide 615 by 5 without using short division?

I have £615 and share it equally between 5 bank accounts. How much will be in each account?

615 pupils need to be put into 5 groups. How many will be in each group?

$$5 \overline{)615}$$

$$615 \div 5 =$$

$$= 615 \div 5$$

What is the calculation?

What is the answer?

